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Experimental Measurement of Neutron Flux and Its Mathematical Data Processing

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Abstract

This paper presents experimental measurement of neutron flux from an AmBe source using a stilbene organic scintillator and subsequent mathematical processing of the measured data. The neutron energy spectrum was reconstructed by solving a Fredholm integral equation of the first kind using maximum likelihood expectation maximization (MLEM) and a novel B-spline-based MLEM-BS method with regularization. Bootstrap resampling was applied to estimate confidence intervals. Results demonstrate successful neutron spectrum unfolding, reduced noise sensitivity of the spline approach, and good agreement of the reconstructed spectrum with the ISO reference standard.

KEY WORDS: *neutron spectroscopy, deconvolution, AmBe source.*

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1. Introduction

Measurement of neutron radiation is of fundamental importance in military applications, particularly in the areas of radiation protection, detection of nuclear materials, and identification of neutron-emitting sources. A detailed understanding of the neutron energy distribution, obtained through neutron spectrometry, enables accurate assessment of radiation exposure risks to personnel, optimization of shielding strategies, and the design of effective protective measures. These capabilities are essential not only under routine operational conditions but also in environments where radiological or nuclear hazards are present.

In the field of defense and security, neutron spectrometry plays a critical role in counter-proliferation efforts, including the monitoring of illicit trafficking of nuclear materials, the detection of improvised nuclear devices, and the forensic analysis of nuclear and radiological incidents. The ability to rapidly and reliably characterize neutron fields is particularly important in chemical, biological, radiological, and nuclear (CBRN) defense operations, as well as in the protection of critical infrastructure and during international military deployments. Consequently, accurate neutron spectrum measurements significantly contribute to the protection of both military personnel and civilian populations, while simultaneously enhancing situational awareness, decision-making processes, and overall operational readiness of armed forces.

Neutron radiation is most commonly measured using scintillation detection techniques, which represent one of the most effective and widely employed methods for neutron detection [1, 2]. Among organic scintillators, both liquid scintillators and solid crystalline materials, such as stilbene, are extensively utilized in neutron spectrometry [3]. Stilbene scintillators are particularly valued for their excellent pulse shape discrimination (PSD) capability, which enables efficient separation of neutron-induced and gamma-induced events. This property arises from the distinct scintillation decay components associated with recoil protons, produced by neutron interactions, and electrons, generated by gamma-ray interactions. As a result, stilbene significantly improves the accuracy and reliability of neutron spectral measurements in mixed radiation fields. In addition, stilbene exhibits favorable characteristics including good energy resolution, high detection efficiency for fast neutrons, and excellent long-term stability. Owing to these properties, it is widely regarded as a reference material for neutron spectrometry in both research and applied military and security contexts.

In this work, neutron detection using organic scintillators is presented as an application of the Fredholm integral equation of the first kind [4]. Fast neutrons are detected through elastic scattering reactions with hydrogen nuclei (protons) contained in the organic scintillator material. During these interactions, a portion of the neutron kinetic energy is transferred to recoil protons, which subsequently lose their energy within the scintillator medium and generate scintillation light. The intensity of the produced light pulses is directly correlated with the energy deposited by the recoil protons. However, because the detected light output represents a convolution of the incident neutron energy spectrum with the detector response function, the original neutron energy distribution cannot be obtained directly. It is necessary to solve following integral equation

$$g(E_\mathrm{p}) = \int_a^b A(E_\mathrm{p}, E_\mathrm{n}) f(E_\mathrm{n}) \mathrm{d}E_\mathrm{n} \,, \tag{1}$$

where E_p and E_n are the proton and neutron energies respectively, $g(E_p)$ is the measured proton spectrum (neutrons are detected through protons), $A(E_p,E_n)$ is the response function and $f(E_n)$ is the neutron spectrum which has to be found by the solution of this equation.

Therefore, so called unfolding algorithm based on the solution of the Fredholm integral equation is required to reconstruct the neutron energy spectrum from the measured light output spectrum. The combination of organic scintillator detectors and advanced unfolding techniques thus enables accurate determination of neutron energy spectra, which is essential for applications in radiation protection, nuclear security, and fundamental nuclear research.

2. Experiment

In the experiments, an americium-beryllium (AmBe) source was used to generate a mixed radiation field consisting of neutrons and gamma rays. Neutrons are generated by the beryllium after the absorption of an alpha particle emitted from the americium. AmBe source is composed of ^{241}Am and ^{9}Be and emits neutrons according to the following chain reaction: $^{241}\mathrm{Am} \rightarrow {}^{237}\mathrm{Np} + \alpha$, $^{9}\mathrm{Be} + \alpha \rightarrow {}^{12}\mathrm{C}^*$ (or ^{12}C) + n. Cesium (^{137}Cs) source was used to provide a pure gamma-ray field for energy calibration. A cylindrical stilbene scintillator with a diameter of 1 inch served as the detector. The radiation source was mounted on a holder and positioned approximately 4 cm horizontally from the scintillator (see Fig.1). The entire section of the apparatus was placed within shielding.

Fig. 1. The pictures of the scintillator with the photomultiplier (the middle of the left picture) and neutron source shielding (right picture).

The light from the scintillator is detected by a photomultiplier. The anode output signal of the photomultiplier tube was connected to the NGA-01 spectrometric system. The NGA-01 is a two-parameter spectrometric device that has been successfully applied in previous studies for neutron and gamma-ray discrimination and spectral analysis [3, 5]. This fully digitized device can process up to 500 000 impulse responses per second. Used system works with high-speed ADC converters with 500 MS/s (alternatively 1 GS/s) and a size of 12 bits. Field Programmable Gate Area (FPGA) with advanced digital filters and PSD algorithms ensures lossless data processing. This achieves the processing of each impulse response from the detector without the dead time of the digital part. The processing parameters are set depending on the used configuration of the detector with a preamplifier. A high voltage source for the photomultiplier is also integrated directly into the system to eliminate external noise.

3. Mathematical Methods

The equation (1) can be expressed in the following discrete form

$$n_i = \sum_{j=1}^{J} R_{ij}\, x_j \qquad i = 1, \dots, I, \tag{2}$$

where n_i is the binned number of counts (responses from protons), x_j is the measured quantity in the jth group (neutron flux), and R_{ij} is the response matrix. The n_i values have a Poisson distribution. The values x_j can be estimated using the maximum likelihood expectation method (MLEM). The MLEM algorithm is widely used to solve equation (2) and it is based on the following iterative sequence [6]

$$x_j^{(k+1)} = \frac{x_j^{(k)}}{\sum_{i=1}^{I} R_{ij}} \sum_{i=1}^{I} R_{ij} \frac{n_i}{\sum_{j=1}^{J} R_{ij} x_j^{(k)}}. \tag{3}$$

In this work, the solution of equation (1) will be assumed in the form of a spline, which is expressed as a linear combination of B-splines. The MLEM-BS algorithm is then formulated on the B-spline space restricted on the B-spline sieve [7] and the B-spline coefficients b_s can be found using following iterative sequence

$$b_s^{(k+1)} = \frac{b_s^{(k)}}{\sum_{i=1}^{I} (RB)_{is} + \beta \frac{\partial P(b)}{\partial b_s}} \times \sum_{i=1}^{I} (RB)_{is} \frac{n_i}{\sum_{s=1}^{N_s} (RB)_{is} b_s^{(k)}}\,, \tag{4}$$

where N_s is the dimension of the B-spline space, B_{is} is the B-spline matrix and b_s are B-spline coefficients. In the penalty term $\beta P(b)$, β is the penalty parameter and the penalty function $P(b)$ is chosen in the form

$$P(b) = \left\| D^{(2)} b \right\|_2^2 \,, \tag{5}$$

where $D^{(2)}$ is the second derivative matrix. The penalty term was added in order to suppress the influence of noise. The number of iterations and the parameters N_s and β are found by minimization of K_S value

$$K_S = \left| \frac{\sum_{i=1}^{I} \left(n_i - \sum_{S=1}^{N_S} (RB)_{is} b_s^{(k)} \right)^2}{\sum_{i=1}^{I} \sum_{S=1}^{N_S} (RB)_{is} b_s^{(k)}} - 1 \right|. \tag{6}$$

In neutron spectrum unfolding, the bootstrap method is used to estimate confidence intervals (CIs) by generating an ensemble of synthetic response vectors via Poisson resampling. An initial spectrum

$$\boldsymbol{n}^{(0)} = \left\{ n_1^{(0)}, n_2^{(0)}, \dots, n_I^{(0)} \right\} \tag{7}$$

is calculated trough backward reconstruction

$$n_i^{(0)} = \sum_{j=1}^{J} R_{ij} x_i^{(0)} \quad i = 1, \dots, I, \tag{8}$$

where the neutron flux $x^{(0)}$ is obtained by unfolding the measured spectrum. For each bootstrap interaction, a new response vector

$$n^{*(b)} = \left\{ n_1^{*(b)}, n_2^{*(b)}, \dots, n_I^{*(b)} \right\},\; b = 1 \dots N_B \tag{9}$$

is constructed. Each component is sampled independently from a Poisson distribution with a mean equal to the corresponding initial spectrum counts $\boldsymbol{n}^{(0)}$. This procedure preserves the counting statistics of the experimental data while introducing random fluctuations consistent with the measurement uncertainty.

For a given energy bin i, let $x_i^{*(b)}$ denote the bootstrap replicates of the unfolded flux. The $100(1-\alpha)\%$ confidence interval is defined by the $\alpha/2$ and $1-\alpha/2$ quantiles of the bootstrap distribution. A 95% confidence interval corresponds to $\alpha = 0.05$.

4. Results and Discussion

This spectrum has to be deconvoluted to obtain the required neutron energy spectrum using the MLEM-BS method described in previous section. The response matrix was calculated with the previously developed Monte Carlo program NEU-7 [8]. The measured spectrum is shown in Fig. 2.

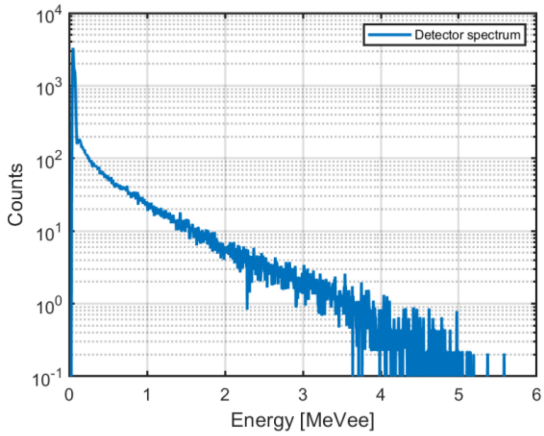

Fig. 2. The energy spectrum obtained from experiment.

The graphs demonstrating the optimization of parameters N_s and β are shown in Fig. 3 and Fig. 4.

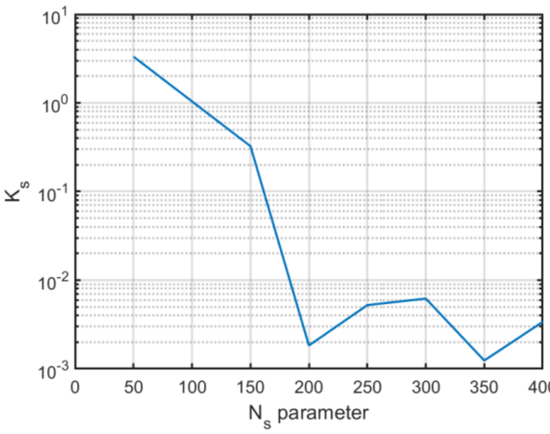

Fig. 3. Optimalization of parameter N_S.

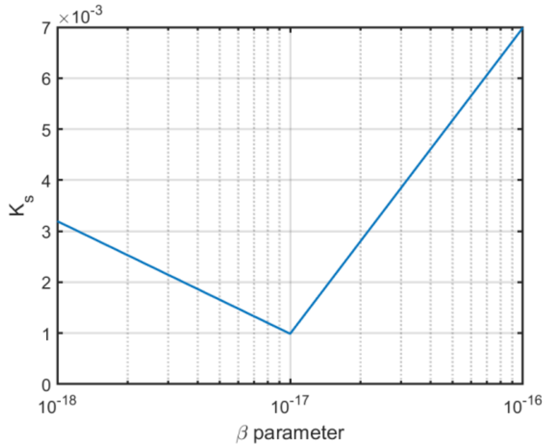

Fig. 4. Optimalization of parameter β.

The numerical experiments with known test data showed that it is suitable to take the parameter N_S which corresponds to the first local minimum. In this case the value of 200 was chosen for the parameter N_S. The resulting neutron energy spectrum is shown in Fig. 4. The results show a good agreement between the results calculated by MLEM and MLEM-BS methods. The spline method suppresses the influence of the noise compared to MLEM method. The neutrons emitted from AmBe source are also reflected by surrounding material back to the scintillator. This effect was included by Monte Carlo simulation of the trajectories of emitted neutrons. It was assumed for this Monte Carlo simulation that the neutron energy spectrum was given by ISO standard. The Monte Carlo simulation showed that the reflection of neutrons leads to increase of neutron energy spectrum at low neutron energies (bellow 2.5 MeV), see the green line in Fig. 6. The confidence interval for calculated neutron energy spectrum is also shown in Fig 6. It can be seen that the confidence interval contains the values of ISO standard.

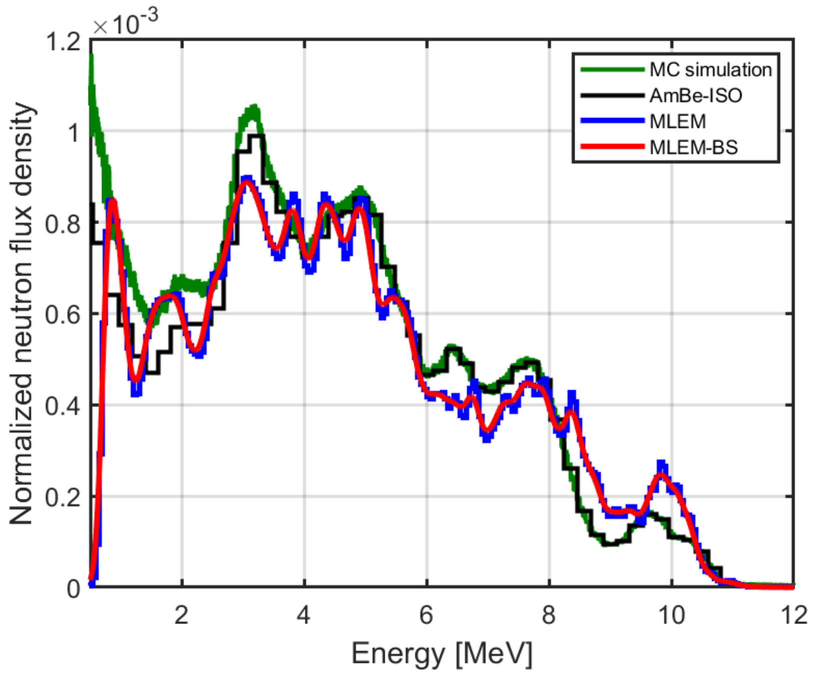

Fig. 5. Results of the deconvolution using the MLEM and MLEM-BS methods for the neutron spectrum from Am-Be source. The B-spline sieve parameters are p = 4, and N_S = 200. The penalty parameter is $\beta = 1.0 \times 10^{-17}$.

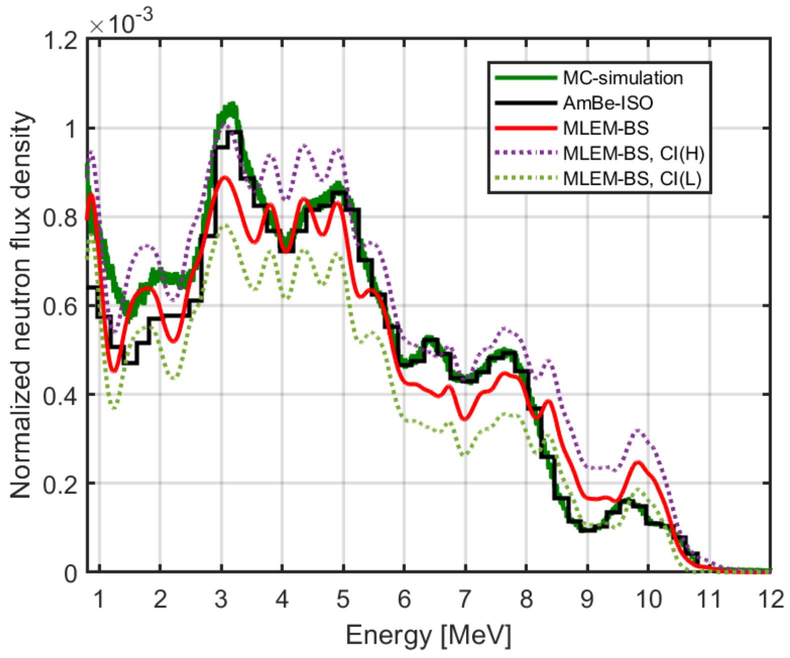

Fig. 6. Confidence interval for MLEM-BS method calculated using boots trap method for the neutron spectrum from Am-Be source.

5. Conclusions

The neutron energy spectrum of an AmBe neutron source was successfully measured using an organic stilbene scintillator detector combined with a fully digitized spectrometric system. The experimental setup enabled reliable acquisition of mixed neutron and gamma radiation data, while pulse shape discrimination allowed effective separation of neutron-induced and gamma-induced events. Mathematical processing of the measured data required deconvolution of the detector response, formulated as the numerical solution of a Fredholm integral equation of the first kind. In addition to the standard MLEM approach, a new deconvolution method based on B-splines with regularization (MLEM-BS) was developed and applied. The proposed spline-based method demonstrated improved suppression of statistical noise while preserving the main characteristics of the neutron spectrum. Confidence intervals of the unfolded spectrum were estimated using bootstrap resampling, which provided an assessment of uncertainty associated with the reconstruction process. The resulting neutron energy spectrum showed good agreement with the ISO reference spectrum for AmBe sources within the estimated confidence bounds. Monte Carlo simulations further confirmed the influence of neutron backscattering from surrounding materials, particularly at lower neutron energies.

The presented methodology confirms that the combination of organic scintillator detectors, advanced unfolding algorithms, and statistical uncertainty analysis provides an effective tool for neutron spectrometry. These results are relevant for applications in radiation protection, nuclear security, CBRN defence, and further research in neutron field characterization. In particular, the ability to accurately reconstruct neutron spectra in mixed radiation environments is essential for dosimetric calculations, detector calibration, and validation of computational transport models. The improved stability of the MLEM-BS algorithm suggests that spline-based regularization may represent a robust alternative to conventional iterative unfolding techniques, especially in cases where measured data are affected by limited counting statistics or strong background contributions.

Furthermore, the developed approach is sufficiently flexible to be adapted to other scintillation detector systems and neutron-emitting radionuclide sources, extending its applicability beyond AmBe calibration fields. Future work may focus on optimization of spline basis parameters, automated selection of regularization strength, and integration of Bayesian inference techniques to further improve spectrum reconstruction accuracy and uncertainty quantification. Additional experimental campaigns in different geometrical arrangements and shielding configurations could also provide deeper insight into environmental effects on measured neutron fields, including room-return contributions and material-dependent scattering phenomena.

Overall, this work contributes to the advancement of neutron spectrometry by combining experimental detector characterization, modern digital signal processing, inverse problem methodology, and rigorous statistical analysis into a unified framework. The proposed methodology offers a practical and computationally efficient solution for neutron spectrum unfolding and may support the development of more accurate neutron monitoring systems in research laboratories, nuclear facilities, emergency response scenarios, and homeland security applications.

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